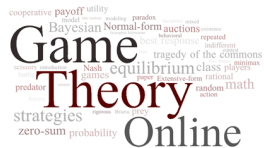




Comparing the Core and Shapley Value in an Example

Game Theory Course:
Jackson, Leyton-Brown & Shoham



Compare Core and Shapley Value in an Example

- UN security council: 15 members.



Compare Core and Shapley Value in an Example

- UN security council: 15 members.
- 5 permanent members: China, France, Russia, UK, US



Compare Core and Shapley Value in an Example



- UN security council: 15 members.
 - 5 permanent members: China, France, Russia, UK, US
 - 10 temporary members

Compare Core and Shapley Value in an Example



- UN security council: 15 members.
 - 5 permanent members: China, France, Russia, UK, US
 - 10 temporary members
 - 5 permanent members can veto resolutions.

Compare Core and Shapley Value in an Example

- UN security council: represent it as a cooperative game.



Compare Core and Shapley Value in an Example



- UN security council: represent it as a cooperative game.
 - China, France, Russia, UK, US are labeled $\{1, 2, 3, 4, 5\}$
 - $v(S) = 1$ if $\{1, 2, 3, 4, 5\} \subset S$ and $\#S \geq 8$,
 - $v(S) = 0$ otherwise.

Compare Core and Shapley Value in an Example

- Let's start with a three-player game that has a similar structure:



Compare Core and Shapley Value in an Example



- Let's start with a three-player game that has a similar structure:
 - 1 permanent member with a veto and 2 temporary members

Compare Core and Shapley Value in an Example



- Let's start with a three-player game that has a similar structure:
 - 1 permanent member with a veto and 2 temporary members
 - $v(S) = 1$ if $1 \in S$ and $\#S \geq 2$,
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Compare Core and Shapley Value in an Example



- $v(S) = 1$ if $1 \in S$ and $\#S \geq 2$, $v(S) = 0$ otherwise.

Compare Core and Shapley Value in an Example



- $v(S) = 1$ if $1 \in S$ and $\#S \geq 2$, $v(S) = 0$ otherwise.

- **Core:** $x_1 + x_2 \geq 1$, $x_1 + x_3 \geq 1$, $x_1 + x_2 + x_3 = 1$, $x_i \geq 0$.

Compare Core and Shapley Value in an Example



- $v(S) = 1$ if $1 \in S$ and $\#S \geq 2$, $v(S) = 0$ otherwise.
- **Core:** $x_1 + x_2 \geq 1$, $x_1 + x_3 \geq 1$, $x_1 + x_2 + x_3 = 1$, $x_i \geq 0$.
- **Core:** $x_1 = 1$, $x_2 = 0$, $x_3 = 0$.

Compare Core and Shapley Value in an Example

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Compare Core and Shapley Value in an Example

- $v(S) = 1$ if $1 \in S$ and $\#S \geq 2$, $v(S) = 0$ otherwise.

- Shapley

$$\text{Value}_i = \frac{1}{N!} \sum_{S \subseteq N \setminus \{i\}} |S|!(|N| - |S| - 1)! [v(S \cup \{i\}) - v(S)].$$



Compare Core and Shapley Value in an Example



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- 1's value: $v(\{1, 2, 3\}) - v(\{2, 3\}) = 1$ weighted by $2/6$,
 $v(\{1, 2\}) - v(\{2\}) = 1$ weighted by $1/6$, $v(\{1, 3\}) - v(\{3\}) = 1$
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Compare Core and Shapley Value in an Example



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weighted by $1/6$
- 2's value: $v(\{1, 2\}) - v(\{1\}) = 1$, weighted by $1/6$

Compare Core and Shapley Value in an Example



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weighted by $1/6$
- 2's value: $v(\{1, 2\}) - v(\{1\}) = 1$, weighted by $1/6$
- 3's value: $v(\{1, 3\}) - v(\{1\}) = 1$, weighted by $1/6$

Compare Core and Shapley Value in an Example



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weighted by $1/6$
- 2's value: $v(\{1, 2\}) - v(\{1\}) = 1$, weighted by $1/6$
- 3's value: $v(\{1, 3\}) - v(\{1\}) = 1$, weighted by $1/6$
- Shapley Value: $x_1 = 2/3$, $x_2 = 1/6$, $x_3 = 1/6$.

A way to the Shapley Value:



Cooperative Games



- Model complex multilateral bargaining and coalition formation, without specifying the particulars of a normal or extensive form
 - Core: Based on coalitional threats - each coalition must get at least what it can generate alone
 - Shapley Value: based on marginal contributions: what does each player contribute to each possible coalition.
 - Other solutions...